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APPLICATION OF HARTLEY DESCRIPTORS

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ABSTRAKT

The paper presents the possible solution of the problem of description of the image object boundary by means of the discrete Hartley transform (DHT). Unlike the discrete Fourier transform (DFT), the DHT kernel is expressed by real numbers. As applied to the real data of the image, it ensures the considerable saving of computational and time resources and the signal processing is simplified.

The method ensuring the reduction of the dimensionality of the input of the classifier of the DHT signs is considered. It provides for even greater practical benefit in the number of operations when processing the images and automatic classification of them. The transform dimensionality reduction can be implemented based on the dispersion of threshold principle of filtering the Hartley transform coefficients (transformants). If the boundary shape has no sharp kinks, the Hartley descriptors are suitable for efficient description of the image object.

The comparative results concerning the assessment of the accuracy of description of the object boundary are presented.

1. INTRODUCTION

The problem of description of the boundaries of the images or identification of individual areas or objects on the images appears when it becomes necessary to describe or identify the individual areas or objects on the images, when the geometry of the objects present on the image are important while the details contained inside the objects or background are of no interest. In certain applications (topography, ecological monitoring, analysis of medical images, etc.) for reducing the volume of array of the data describing the object, it is desirable to replace a set of pixels depicting the object with description of its boundary.

The description of the object (boundaries of the – area after segmentation of the images) is performed on the basis of the point, linear (one-dimensional) and two-dimensional representations. The linear and two-dimensional objects can be presented in the form of closed external and internal contours. In this case the area can be described by signs obtained with the help of the discrete Fourier transform (DFT). The Fourier descriptors serve as attributes participating in the description, comparison with the specific pattern and when separating the images to be classified [1]. However, the DFT has a disadvantage consisting in necessity of operation with complex numbers that cause the doubling of computations. The computational complexity increases still more when the DFT is performed for each independent variable.

The spatial data describing the boundary can have high dimensionality. Then at the stage of obtaining the recognition attributes, the computational and time complexity of the processing increases still more. To simplify the processing and the process of classification of the object pattern, the dimensionality of the classifier input should be reduced. One of methods of reducing the computational complexity consists in elimination of the spatial redundancy and reducing the number of attributes to be analyzed. The dimensionality shall be reduced in such a way that the concomitant increase of the recognition error would be relatively small. As is known, the Karhunen-Loeve decorrelation transformation (KLT) is the optimum one among all the linear transformation in respect of the criterion of the minimum value of the root-mean-square error in case of incomplete dimensionality (inaccurate specification of coordinates). However, the KLT has no fast computation algorithms that hampers its practical use. In this paper, the possibility of using the attributes of the discrete Hartley transform for describing the object boundary is investigated.

2. HARTLEY DESCRIPTORS

Unlike the DFT, the Hartley orthogonal basis specified on the range of N points 0, 1, 2,…, $N-1$, is expressed by real numbers [2]

$$h_h = \operatorname{cas}\left(\frac{2\pi v n}{N}\right) = \cos\left(\frac{2\pi v n}{N}\right) + \sin\left(\frac{2\pi v n}{N}\right), \quad (1)$$

$n, v \in \{0,1,\dots,N-1\}$.

For example, the matrix $\boldsymbol{H}_h$ of the discrete set of the orthogonal functions of the DHT with the dimension of 8×8 looks as follows:

$$\boldsymbol{H}_h = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \sqrt{2} & 1 & 0 & -1 & -\sqrt{2} & -1 & 0 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 0 & -1 & \sqrt{2} & -1 & 0 & 1 & -\sqrt{2} \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -\sqrt{2} & 1 & 0 & -1 & \sqrt{2} & -1 & 0 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 0 & -1 & -\sqrt{2} & -1 & 0 & 1 & \sqrt{2} \end{bmatrix}.$$

The pair 1-D DHT of the sequence $\{g_n\} = \{g_0, g_1, \dots, g_{N-1}\}$ is determined by the following equations:

$$\hat{g}_v = \sum_{n=0}^{N-1} g_n \operatorname{cas}\left(\frac{2\pi v n}{N}\right), \quad (2)$$

$$g_n = \frac{1}{N}\sum_{k=0}^{N-1} \hat{g}_v \operatorname{cas}\left(\frac{2\pi v n}{N}\right), \quad (3)$$

where $\{\hat{g}_v\} = \{\hat{g}_0, \hat{g}_1, \dots, \hat{g}_{N-1}\}$ is the Hartley pattern of the sequence of counts $\{g_n\}$.

In the matrix representation, the operations of the 1D DHT of the vectors $\boldsymbol{g}$ и $\hat{\boldsymbol{g}}$ are expressed as

$$\hat{\boldsymbol{g}} = \boldsymbol{H}_h \boldsymbol{g} \text{ and } \boldsymbol{g} = \frac{1}{N}\boldsymbol{H}_h \hat{\boldsymbol{g}}, \quad (4)$$

where $\boldsymbol{g}$ is the column-vector of the discrete values of the signal with the dimension $N \times 1$; $\hat{\boldsymbol{g}}$ is the column-vector of the of the spectral DHT with the dimension of $N \times 1$. As is seen, the direct Hartley transform and the inverse one have no differences; they are mutually symmetrical.

2.1. Description of the Boundary

Let's consider that the initial halftone image has been replaced with the binary one. Let the discrete boundary g of the area contains N counts (points). It can be presented as a two-dimensional function

$$g_{x_n, y_n}, n \in \{0, 1, \dots, i, \dots, j, \dots N-1\},$$

where (x_i, y_j) are integer pairs of the Cartesian product Z^2. The function g_{x_i, y_j} corresponds definitely to each point of the boundary contour. The whole contour is presented by a point sequence

$$\{g_{x_i, y_j}\} = \{(x_0, y_0), \dots, (x_i, y_j) \dots, (x_{N-1}, y_{N-1}\}. \quad (5)$$

Now we shall form two one-dimensional sequences (vectors) from the coordinates of the sequence $\{g_{x_n, y_n}\}$:

$x_n = \{x_0, x_1, \dots, x_{N-1}\}$ und $y_n = \{y_0, y_1, \dots, y_{N-1}\}$.

With the account the representation (4), the attributes in the Hartley expansion: $\hat{\boldsymbol{g}}_x$ and $\hat{\boldsymbol{g}}_y$ correspond definitely to the vectors $\boldsymbol{g}_x$ and $\boldsymbol{g}_y$. These vectors represent of patterns of the object boundaries. Such record of the two-dimensional boundary g of the area makes it possible to reduce the computational expenses for its description.

Reduction of the dimensionality of the classifier input is implemented by presenting efficiently the sequences $\{x_n\}$ und $\{y_n\}$ in the spectral DHT basis. The solution of this problem is possible on the basis of the dispersion or threshold methods of filtering the Hartley transform coefficients [3]. To do this, the transformants with the maximum dispersions are extracted from the vectors of the patterns $\hat{\boldsymbol{g}}_x$ and $\hat{\boldsymbol{g}}_y$. In the methods of recognition of binary images based on the comparison, the unknown pattern is ascribed to the class, the prototype of which is the nearest in terms of the preliminarily selected metric [4]. The subset of attributes, which is selected after reducing the dimensionality, shall correspond to the vector of the attributes of the boundary pattern proceeding from, for example, criterion of the minimum distance between them. It is supposed that the less is the distance between the objects being compared, the more is their similarity degree.

2.1.1. Examples of Application of the DHT

Example 1.

Let's consider a closed contour (boundary) with the number of points of the binary image $N = 16$ as an illustration of application of the Hartley descriptors. The example of the discrete boundary is shown in Figure 1.

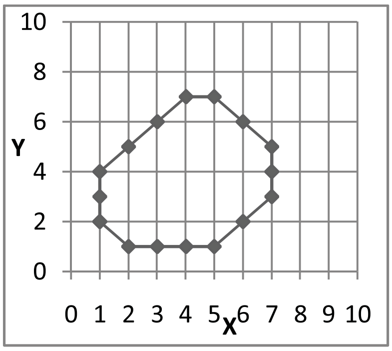


Figure 1 16-point boundary of the object

The two-dimensional sequence
$\{g\} = \{(x_0, y_0), (x_1, y_1), \dots, (x_{15}, y_{15})\} =$
$= \{(1,2)(1,3)(1,4)(2,5)(3,6)(4,7)(5,7)(6,6)$
$(7,5)(7,4)(7,3)(6,2)(5,1)(4,1)(3,1)(2,1)\}$
corresponds to the boundary points.

The initial one-dimensional sequences are
$\{x_n\} = \{1, 1, 1, 2, 3, 4, 5, 6, 7, 7, 7, 6, 5, 4, 3, 2\}$ and
$\{y_n\} = \{2, 3, 4, 5, 6, 7, 7, 6, 5, 4, 3, 2, 1, 1, 1, 1\}$.

The DHT of the sequences $\{x_n\}$ and $\{y_n\}$. is performed according to formula (2). As a result, we obtain the following sequences of the attributes presenting the initial coordinates of the observation object:

$\{\hat{g}_{v_x}\} = \{64; -31{,}72; 0; -1{,}64; 0; 0{,}3; 0; -0{,}5;$
$0; -1{,}2; 0; -0{,}72; 0; 0{,}62; 0; -13{,}14\}$.

$\{\hat{g}_{v_y}\} = \{58; 6{,}57; -2{,}41; 1{,}16; 0; 0{,}36; 1; 0{,}11;$
$0; 0{,}25; 0{,}41; -0{,}24; -2; 0{,}82; 1; -33{,}03\}$.

To reduce the dimensionality of the field of attributes the function (vector) of distribution of the dispersion of the coefficients $\{\hat{g}_{v_x}\}$ and $\{\hat{g}_{v_y}\}$ is calculated [3]. The coefficients of the vector with the minimum values determine the zone of filtering the transformants or that of saving M attributes. For the sequences $\{\hat{g}_{v_x}\}$ and $\{\hat{g}_{v_y}\}$, these values are $M_x = 3$ и $M_y = 3$, respectively. The filtering zone includes the coefficients $\{\hat{g}_{v_x}\}$ and $\{\hat{g}_{v_y}\}$, the coordinates of which correspond to the set

$$\{L\} = \{(x_2, y_2), (x_3, y_3), \dots, (x_{14}, y_{14})\}.$$

In our case the address information about the zone of filtering the transformants is only determined by the two extreme coordinates of the set $\{L\}$. Abandon of transmission of the information about all the transformants filtered improves the efficiency of the data processing as a whole. The truncated input sequences are equal to

$$\{\hat{g}'_{v_x}\} = \{64; -31{,}72; -13{,}14\},$$

$$\{\hat{g}'_{v_y}\} = \{58; 6{,}57; -33{,}03\}.$$

The image of the area boundaries will be restored using M saved coefficients. By applying the formula (3) for truncated sequences, the following estimates of $\{\tilde{x}_n\}$ and $\{\tilde{y}_n\}$ will be obtained:

$$\{\tilde{x}_n\} = \{1{,}96; 0{,}96; 1{,}2; 1{,}85; 2{,}84; 4; 5{,}16; 6{,}15; 6{,}8; 7{,}04; 6{,}8; 6{,}15; 5{,}16; 4; 2{,}84; 1{,}85\}.$$

$$\{\tilde{y}_n\} = \{1{,}97; 3{,}04; 4{,}2; 5{,}28; 6{,}1; 6{,}55; 6{,}53; 6{,}1; 5{,}28; 4{,}2; 3{,}05; 1{,}97; 1{,}15; 0{,}7; 1{,}14\}.$$

After performing the rounding operation, we have the values of the coordinates of the object boundary, i.e.

$$\{\tilde{g}\} = \{g_{x_i y_j}\} = \text{round}\{g_{\tilde{x}_n \tilde{y}_n}\} = \\ = \{(1,2)(1,3)(1,4)(2,5)(3,6)(4,7)(5,7)(6,6) \\ (7,5)(7,4)(7,3)(6,2)(5,1)(4,1)(3,1)(2,1)\}.$$

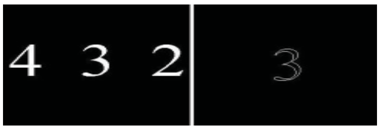
If $\{\tilde{g}_c\} = \{\tilde{x}_n\} \circ \{\tilde{y}_n\}$ is an estimate of $\{g_c\} = \{x_n\} \circ \{y_n\}$, the root-mean-square error of restoration of the boundary points is

$$e_{RMSE} = \sqrt{\frac{1}{2N}\sum_{i=0}^{N-1}(\tilde{g}_{ci} - g_{ci})^2}.$$

It is obvious that the distortion value in the example being considered $e_{RMSE} = 0$. As is seen, relatively small space of the saved DHT attributes describe the object precisely (6 attributes determine precisely the irregularly shaped curve). If 4 of 16 attributes are saved, the root-mean-square error of restoration of the boundary points does not exceed the value $e_{RMSE} = 0{,}75$.

Exampl 2.

The binary images of digits, Figure 2a, were selected as initial data for performing the processing simulation process. Figure 2(b) shows the binary image of the 456-point boundary of the digit 3.



a) b)

Figure 2(a) Image of digits

Figure 2(b) Image of the boundary of the digit 3

Figure 3 shows the restored images of the digit 3. When describing the digit, 30, 25, 20, 15, 10 and 8 Hartley descriptors, respectively, were used. It corresponds to 6.6%, 5.5%, 4.4%, 3.3%, 2.2% and 1.8% of all 456 descriptors, respectively.

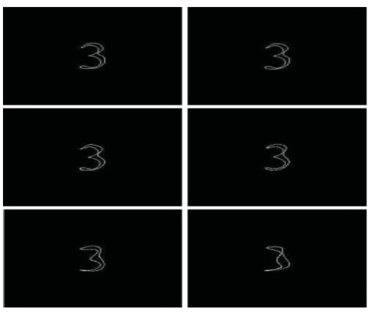


Figure 3 Images of the restored boundaries of the digit 3 with using the Hartley descriptors

To compare the efficiency of description of the boundaries with the help of DHT and DFT.

Fig. 4 presents the images of the restored boundaries of the digit 3 by processing the Fourier descriptors. When describing the digit, 30, 25, 20, 15, 10 and 8 Fourier descriptors, respectively, were used. It corresponds to 6.6%, 5.5%, 4.4%, 3.3%, 2.2% and 1.8% of all 456 descriptors, respectively.

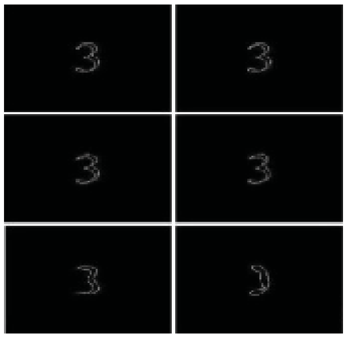


Figure 4 Images of the restored boundaries of the digit 3 with applying the Fourier descriptors

Figure 5 presents the dependence of the root mean square error of restoring the boundary points on the number of Hartley (the Fourier) descriptors of the digit 3 to be saved.

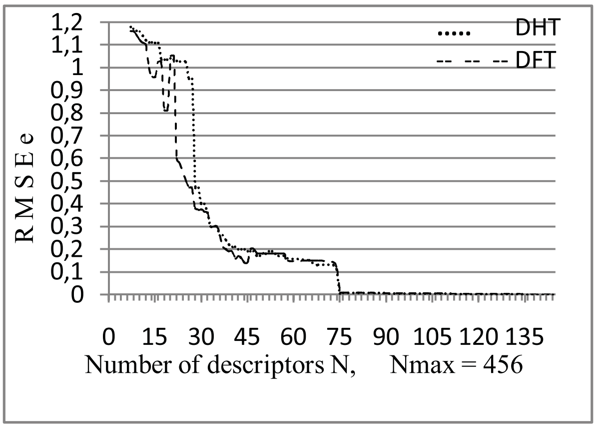


Figure 5 Dependence of the root-mean-square error on the number of Hartley (the Fourier) attributes

As follows from the presented comparison, the root-mean-square errors of restoring the boundaries using the DHT and DFT methods, respectively, have practically the same value. If more than 74 descriptors are considered, the value = 0 , i.e. there are no distortions and the object shape is identical to the initial pattern.

As the input size N (number of points of the boundary) increases, the efficiency of processing increases too. The description complexity reduces due to use of the fast Hartley algorithm.

3. CONCLUSIONS

The following capabilities of the DHT are implemented as applied to the real data:

Hartley descriptors make it possible to:

1) present efficiently the boundaries of the image object in a compact form;
2) restore the boundary image with the controllable error;
3) simplify the solving of the problems of classification, identification and recognition of patterns.

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